

# Cahier de calcul

— réponses —



*Margarita philosophica* (La perle philosophique), Gregor REISCH (1508)

Cette gravure, extraite d'un manuel d'université de l'époque, représente Arithmetica, allégorie des mathématiques, arbitrant une compétition entre Boèce, qui utilise les chiffres indo-arabes, et Pythagore, qui utilise un boulier.

Ce cahier de calcul a été écrit collectivement.

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# Fiche n° 1. Fractions

## Réponses

- |             |                      |             |                         |              |                                   |
|-------------|----------------------|-------------|-------------------------|--------------|-----------------------------------|
| 1.1 a)..... | $\frac{4}{5}$        | 1.3 c)..... | $\frac{-10}{3}$         | 1.7 .....    | $\frac{n^3 + n}{n + 1}$           |
| 1.1 b)..... | $2^5$                | 1.3 d)..... | 1 000                   | 1.8 a).....  | $4 + \frac{5}{6}$                 |
| 1.1 c)..... | 3                    | 1.4 .....   | $\frac{16}{35}$         | 1.8 b).....  | $1 + \frac{1}{k - 1}$             |
| 1.1 d)..... | $-2 \times 3^{3k-2}$ | 1.5 a)..... | 2 022                   | 1.8 c).....  | $3 + \frac{5}{x - 2}$             |
| 1.2 a)..... | $\frac{1}{6}$        | 1.5 b)..... | $\frac{1}{2}$           | 1.9 .....    | 2t                                |
| 1.2 b)..... | $\frac{7}{15}$       | 1.5 c)..... | 1                       | 1.10 a)..... | $\frac{3}{5} > \frac{5}{9}$       |
| 1.2 c)..... | 9                    | 1.5 d)..... | 2                       | 1.10 b)..... | $\frac{12}{11} > \frac{10}{12}$   |
| 1.2 d)..... | $\frac{1}{9}$        | 1.6 a)..... | $\frac{-1}{n(n + 1)^2}$ | 1.10 c)..... | $\frac{125}{25} = \frac{105}{21}$ |
| 1.3 a)..... | 247                  | 1.6 b)..... | $-\frac{ab}{a - b}$     | 1.11 .....   | Non                               |
| 1.3 b)..... | $\frac{203}{24}$     | 1.6 c)..... | $\frac{3}{2}n$          | 1.12 .....   | A > B                             |

## Fiche n° 2. Puissances

### Réponses

2.1 a).....  $10^8$

2.1 b).....  $10^{15}$

2.1 c).....  $10^2$

2.1 d).....  $10^{-2}$

2.1 e).....  $10^4$

2.1 f).....  $10^{-8}$

2.2 a).....  $15^4$

2.2 b).....  $5^{-6}$

2.2 c).....  $2^7$

2.2 d).....  $(-7)^{-2}$

2.2 e).....  $3^5$

2.2 f).....  $3^{28}$

2.3 a).....  $2^{-4} \cdot 3^{-1}$

2.3 b).....  $2^{21} \cdot 3$

2.3 c).....  $2$

2.3 d).....  $2^{38} \cdot 3^{26}$

2.4 a).....  $8$

2.4 b).....  $11$

2.4 c).....  $3^{10}$

2.4 d).....  $2^6 \cdot 5$

2.5 a).....  $\frac{x}{x+1}$

2.5 b).....  $\frac{1}{x-2}$

2.5 c).....  $\frac{2x}{x+1}$

2.5 d).....  $\frac{2}{x-2}$

## Fiche n° 3. Calcul littéral

### Réponses

3.1 a) .....  $8x^3 - 6x^2 + \frac{3}{2}x - \frac{1}{8}$

3.1 b) .....  $x^5 - 2x^4 + x^3 - x^2 + 2x - 1$

3.1 c) .....  $x^5 - x^3 + x^2 - 1$

3.1 d) .....  $x^5 + 2x^4 + x^3 - x^2 - 2x - 1$

3.1 e) .....  $x^5 - x^3 - x^2 + 1$

3.1 f) .....  $x^4 + x^2 + 1$

3.2 a) .....  $-2 + 12x - 17x^2 + 8x^3 - 3x^4$

3.2 b) .....  $-28 + 21x$

3.2 c) .....  $2 + x^3 - x^4 - x^5$

3.2 d) .....  $-1 - 3x - 3x^2 + x^3$

3.2 e) .....  $1 + x^4$

3.2 f) .....  $1 + 2x + 3x^2 + 2x^3 + x^4$

3.3 a) .....  $-6(6x + 7)$

3.3 b) .....  $4(5x + 4)(-5x + 1)$

3.3 c) .....  $2(3x - 4)(10x + 3)$

3.3 d) .....  $-8(x + 1)(x + 16)$

3.4 a) .....  $(x - 1)^2$

3.4 b) .....  $(x + 2)^2$

3.4 c) .....  $(x + 1)(x + 2)$

3.4 d) .....  $3\left(x + \frac{7 - \sqrt{37}}{6}\right)\left(x + \frac{7 + \sqrt{37}}{6}\right)$

3.4 e) .....  $2\left(x + \frac{3 - \sqrt{233}}{4}\right)\left(x + \frac{3 + \sqrt{233}}{4}\right)$

3.4 f) .....  $-5(x - 1)\left(x - \frac{1}{5}\right)$

3.5 a) .....  $(x + y - z)(x + y + z)$

3.5 b) .....  $3(14x + 3y)(-4x + y)$

3.5 c) .....  $(x + 1)(y + 1)$

3.5 d) .....  $(x - 1)(y - 1)$

3.5 e) .....  $(x + y)(x + 1)^2$

3.5 f) .....  $(a^2 + b^2)(y - 4x^2)(y + 4x^2)$

3.6 a) .....  $(x - 1)(x + 1)(x^2 + 1)$

3.6 b) .....  $-8(x^2 + 1)(x - 4)(x + 4)$

3.6 c) .....  $(x^2 + x + 1)(x^2 - x + 1)$

3.6 d) .....  $(a^2 + b^2)(c^2 + d^2)$

3.6 e) .....  $(a^2 + b^2 + c^2 + d^2)(p^2 + q^2 + r^2 + s^2)$

## Fiche n° 4. Racines carrées

### Réponses

4.1 a).....	$5$	4.3 a) ....	$2 - \sqrt{2} - \sqrt{3} + \frac{1}{2}\sqrt{6}$	4.5 c).....	$1 + \sqrt{x-1}$
4.1 b).....	$\sqrt{3} - 1$	4.3 b) .....	$3 - 2\sqrt{2}$	4.5 d) .....	$\frac{1}{2} \frac{1}{x-1}$
4.1 c).....	$-\sqrt{3} + 2$	4.3 c).....	$1 - \sqrt{10} + \sqrt{15}$	4.5 e) .....	$\frac{x(x-2)}{(x-1)\sqrt{x-1}}$
4.1 d).....	$\sqrt{7} - 2$	4.3 d)....	$\sqrt{15} + \sqrt{10} - \sqrt{6} - 2$	4.5 f) .....	$-4(x-1)^2$
4.1 e).....	$\pi - 3$	4.3 e) .....	$-(\sqrt{2} + \sqrt{3})$	4.6 a).....	$\sqrt{2}$
4.1 f).....	$ 3 - a $	4.3 f)....	$-\frac{3 + \sqrt{2} + \sqrt{3} + \sqrt{6}}{2}$	4.6 b) .....	$2\sqrt{2}$
4.2 a) .....	$20$	4.3 g).....	$2\sqrt{2}$	4.7 a).....	$-11 + 5\sqrt{5}$
4.2 b) .....	$9 + 4\sqrt{5}$	4.3 h) .....	$50 - 25\sqrt{3}$	4.7 b).....	$1 + \sqrt{2}$
4.2 c).....	$1 + \sqrt{3}$	4.4 .....	$\frac{\sqrt{2} + 2 - \sqrt{6}}{4}$	4.7 c).....	$1 + \sqrt{2}$
4.2 d).....	$3 + \sqrt{2}$	4.5 a).....	$\frac{x}{\sqrt{x-1}}$	4.7 d).....	$\sqrt{3}$
4.2 e).....	$12\sqrt{7}$	4.5 b).....	$x - \sqrt{x^2 - 1}$	4.7 e).....	$1 + \sqrt{5}$
4.2 f).....	$12$			4.7 f) .....	$\ln(1 + \sqrt{2})$
4.2 g).....	$9 - \frac{10}{3}\sqrt{2}$			4.8 .....	$1$
4.2 h) .....	$10$				



## Fiche n° 5. Expressions algébriques

### Réponses

5.1 a) .....  $7a^2 + 12a + 7$

5.1 b) .....  $a^2 - 1$

5.1 c) .....  $4a^2 - a - 3$

5.1 d) .....  $-a^2 + 1$

5.2 a) .....  $8 + 6i$

5.2 b) .....  $8 - 6i$

5.2 c) .....  $18 - 26i$

5.2 d) .....  $-9 - 46i$

5.3 a) .....  $39 - 18i$

5.3 b) .....  $2197$

5.3 c) .....  $-4 + 43i\sqrt{5}$

5.3 d) .....  $1$

5.4 a) .....  $3$

5.4 b) .....  $1$

5.4 c) .....  $1$

5.4 d) .....  $0$

5.4 e) .....  $-1$

5.4 f) .....  $31$

5.5 a) .....  $a^2 + 2$

5.5 b) .....  $a^3 + 3a$

5.5 c) .....  $a^4 + 4a^2 + 2$

5.6 a) .....  $a^2 - 2b$

5.6 b) .....  $ab - 3c$

5.6 c) .....  $a^3 - 3ab + 3c$

5.6 d) .....  $ab - c$

5.6 e) .....  $ac$

5.6 f) .....  $-2ac + b^2$

5.7 a) .....  $a^2b - ac - 2b^2$

5.7 b) ...  $a^4 - 4a^2b + 4ac + 2b^2$

5.7 c) .....  $0$

5.7 d) .....  $1$

5.7 e) .....  $a$

## Fiche n° 6. Équations du second degré

### Réponses

- 6.1 a) .....  $3, 3$
- 6.1 b) .....  $-1/3, -1/3$
- 6.1 c) .....  $2, -6$
- 6.1 d) .....  $2, 3$
- 6.1 e) .....  $0, \text{ donc } 5$
- 6.1 f) .....  $0, \text{ donc } -3/2$
- 6.1 g) .....  $\emptyset$
- 6.1 h) .....  $1 \text{ donc } -5$
- 6.1 i) .....  $1 \text{ donc } 8/3$
- 6.1 j) .....  $-1 \text{ donc } -19/5$
- 6.2 a) .....  $6, 7$
- 6.2 b) .....  $-3, -5$
- 6.2 c) .....  $-7, -11$
- 6.2 d) .....  $-3, 11$
- 6.2 e) .....  $a, b$
- 6.2 f) .....  $a - b, a + b$
- 6.3 a) .....  $2/3$
- 6.3 b) .....  $-2/7$
- 6.3 c) .....  $-1/m$
- 6.3 d) .....  $2m/(m + 3)$
- 6.4 a) .....  $1 \text{ donc } (a - b)/(b - c)$
- 6.4 b) .....  $1 \text{ donc } c(a - b)/(a(b - c))$
- 6.4 c) .....  $m \text{ donc } -(m + a + b)$
- 6.4 d) .....  $m \text{ donc } m(a - b)/(b - c)$
- 6.4 e) .....  $m \text{ donc } ab/m$
- 6.4 f) .....  $a + b \text{ puis } 2ab/(a + b).$
- 6.5 a) .....  $x^2 - 22x + 117 = 0$
- 6.5 b) .....  $x^2 - 6x - 187 = 0$
- 6.5 c) .....  $x^2 - 4x + 1 = 0$
- 6.5 d) .....  $x^2 - 2mx + 3 = 0$
- 6.5 e) .....  $2x^2 - (4m + 1)x + (2m^2 + m - 15) = 0$
- 6.5 f) .....  $m^2x^2 + (m - 2m^2)x + (m^2 - m - 2) = 0$
- 6.6 a) .....  $m = -3/4 \text{ et } x = 3/4$
- 6.6 b) ...  $m = -1 \text{ et } x = -2, \text{ ou } m = 7 \text{ et } x = 2/3$
- 6.6 c) .....  $m = 1 \text{ et } x = -1 \text{ ou } m = -1 \text{ et } x = 1$
- 6.7 a) .....  $a = 2 \text{ et } b = 3$
- 6.7 b) .....  $a = -2 \text{ et } b = 1$
- 6.7 c) .....  $a = -3 \text{ et } b = 5$
- 6.7 d) .....  $a = 1/2 \text{ et } b = 8$
- 6.7 e) .....  $a = 1 \text{ et } b = 3\sqrt{7}$
- 6.8 a) .....  $] -\infty, 1] \cup [\sqrt{2}, +\infty[$
- 6.8 b) .....  $[-3, 5]$
- 6.8 c) .....  $] -\infty, -1] \cup [2/3, +\infty[$
- 6.8 d) .....  $] -\infty, -1/2[ \cup [4, +\infty[$

# Fiche n° 7. Exponentielle et logarithme

## Réponses

7.1 a).....	$4 \ln 2$	7.5 b).....	$\frac{1}{2}$	7.8 a).....	$\mathbb{R}$
7.1 b).....	$9 \ln 2$	7.5 c).....	$\frac{1}{3}$	7.8 b).....	ok
7.1 c).....	$-3 \ln 2$	7.5 d).....	$\frac{1}{9}$	7.8 c).....	1
7.1 d).....	$\frac{1}{2} \ln 2$	7.5 e).....	$-\frac{1}{2}$	7.8 d).....	-1
7.1 e).....	$3 \ln 2$	7.5 f).....	$\frac{3}{2}$	7.9 a).....	$x + \ln 2$
7.1 f).....	$2 \ln 2 + 2 \ln 3$	7.6 a).....	-2	7.9 b).....	$\frac{e^x}{\sqrt{1+x}}$
7.2 a).....	$-\ln 3 - 2 \ln 2$	7.6 b).....	$\frac{1}{\ln 2}$	7.9 c).....	$\ln  x - 1 $
7.2 b).....	$2 \ln 3 - 2 \ln 2$	7.6 c).....	-17	7.9 d).....	$-\frac{1}{1+x}$
7.2 c).....	$\ln 3 + 11 \ln 2$	7.6 d).....	1	7.9 e).....	$(1+x)^x$
7.2 d).....	$3 \ln 5 + 2 \ln 2$	7.6 e).....	-1	7.10 a).....	$x \geq \frac{\ln 12 + 5}{3}$
7.2 e).....	$-2 \ln 5 + 4 \ln 2$	7.6 f).....	e	7.10 b).....	$x \in [0, 1]$
7.2 f).....	$2 \ln 5 - 2 \ln 2$	7.7 a).....	impaire	7.10 c).....	$x \geq \frac{2}{e}$
7.3.....	$-2 \ln 2 - 2 \ln 5$	7.7 b).....	impaire	7.10 d).....	$x \geq -\frac{1}{12}$
7.4 a).....	$\frac{25}{8} \ln(\sqrt{2} - 1)$	7.7 c).....	impaire	7.10 e).....	$\emptyset$
7.4 b).....	$17 + 12\sqrt{2}$	7.7 d).....	impaire	7.10 f).....	$\frac{-13 - \sqrt{273}}{2}$
7.4 c).....	0				
7.4 d).....	0				
7.5 a).....	8				

# Fiche n° 8. Trigonométrie

## Réponses

8.1 a).....  $0$

8.1 b).....  $0$

8.1 c).....  $-1 - \sqrt{3}$

8.1 d).....  $-\frac{1}{2}$

8.2 a).....  $0$

8.2 b).....  $-\sin x$

8.2 c).....  $2 \cos x$

8.2 d).....  $-2 \cos x$

8.3 a).....  $\frac{\sqrt{6} - \sqrt{2}}{4}$

8.3 b).....  $\frac{\sqrt{6} + \sqrt{2}}{4}$

8.3 c).....  $\frac{\sqrt{6} - \sqrt{2}}{4}$

8.3 d).....  $\frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

8.4 a).....  $-\sin x$

8.4 b).....  $\frac{1}{\cos x}$

8.4 c).....  $0$

8.4 d).....  $4 \cos^3 x - 3 \cos x$

8.5 a).....  $\frac{\sqrt{2} + \sqrt{2}}{2}$

8.5 b).....  $\frac{\sqrt{2} - \sqrt{2}}{2}$

8.6 a).....  $\tan x$

8.6 b).....  $2$

8.6 c).....  $8 \cos^4 x - 8 \cos^2 x + 1$

8.7 a).....  $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

8.7 a).....  $\left\{ -\frac{\pi}{3}, \frac{\pi}{3} \right\}$

8.7 a).....  $\left\{ \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$

8.7 b).....  $\left\{ \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

8.7 b).....  $\left\{ \frac{-2\pi}{3}, \frac{-\pi}{3} \right\}$

8.7 b).....  $\left\{ \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$

8.7 c).....  $\left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

8.7 c).....  $\left\{ -\frac{5\pi}{6}, -\frac{\pi}{6} \right\}$

8.7 c).....  $\left\{ \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\}$

8.7 d).....  $\left\{ \frac{3\pi}{4}, \frac{7\pi}{4} \right\}$

8.7 d).....  $\left\{ -\frac{\pi}{4}, \frac{3\pi}{4} \right\}$

8.7 d).....  $\left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}$

8.7 e).....  $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

8.7 e).....  $\left\{ -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \right\}$

8.7 e).....  $\left\{ \frac{\pi}{4} + k\frac{\pi}{2}, k \in \mathbb{Z} \right\}$

8.7 f).....  $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

8.7 f).....  $\left\{ -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

8.7 f).....  $\left\{ \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + k\pi, k \in \mathbb{Z} \right\}$

8.7 g).....  $\left\{ \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12} \right\}$

8.7 g).....  $\left\{ -\frac{11\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{11\pi}{12} \right\}$

8.7 g).....  $\left\{ \frac{\pi}{12} + k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{11\pi}{12} + k\pi, k \in \mathbb{Z} \right\}$

8.7 h).....  $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

8.7 h).....  $\left\{ -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

- 8.7 h) .....  $\left\{ \frac{\pi}{6} + k \frac{2\pi}{3}, k \in \mathbb{Z} \right\}$
- 8.7 i) .....  $\left\{ \frac{\pi}{7}, \frac{13\pi}{7} \right\}$
- 8.7 i) .....  $\left\{ -\frac{\pi}{7}, \frac{\pi}{7} \right\}$
- 8.7 i) .....  $\left\{ \frac{\pi}{7} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z} \right\}$
- 8.7 j) .....  $\left\{ \frac{5\pi}{14}, \frac{9\pi}{14} \right\}$
- 8.7 j) .....  $\left\{ \frac{5\pi}{14}, \frac{9\pi}{14} \right\}$
- 8.7 j) .....  $\left\{ \frac{5\pi}{14} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{9\pi}{14} + 2k\pi, k \in \mathbb{Z} \right\}$
- 8.8 a) .....  $\left[ 0, \frac{3\pi}{4} \right] \cup \left[ \frac{5\pi}{4}, 2\pi \right]$
- 8.8 a) .....  $\left[ -\frac{3\pi}{4}, \frac{3\pi}{4} \right]$
- 8.8 b) .....  $\left[ \frac{\pi}{3}, \frac{5\pi}{3} \right]$
- 8.8 b) .....  $\left[ -\pi, -\frac{\pi}{3} \right] \cup \left[ \frac{\pi}{3}, \pi \right]$
- 8.8 c) .....  $\left[ 0, \frac{\pi}{6} \right] \cup \left[ \frac{5\pi}{6}, 2\pi \right]$
- 8.8 c) .....  $\left[ -\pi, \frac{\pi}{6} \right] \cup \left[ \frac{5\pi}{6}, \pi \right]$
- 8.8 d) .....  $\left[ 0, \frac{\pi}{6} \right] \cup \left[ \frac{5\pi}{6}, \frac{7\pi}{6} \right] \cup \left[ \frac{11\pi}{6}, 2\pi \right]$
- 8.8 d) .....  $\left[ -\pi, -\frac{5\pi}{6} \right] \cup \left[ -\frac{\pi}{6}, \frac{\pi}{6} \right] \cup \left[ \frac{5\pi}{6}, \pi \right]$
- 8.8 e) .....  $\left[ \frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[ \frac{5\pi}{4}, \frac{3\pi}{2} \right]$
- 8.8 e) .....  $\left[ -\frac{3\pi}{4}, -\frac{\pi}{2} \right] \cup \left[ \frac{\pi}{4}, \frac{\pi}{2} \right]$
- 8.8 f) .....  $\left[ \frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[ \frac{\pi}{2}, \frac{3\pi}{4} \right] \cup \left[ \frac{5\pi}{4}, \frac{3\pi}{2} \right] \cup \left[ \frac{3\pi}{2}, \frac{7\pi}{4} \right]$
- 8.8 f) .....  $\left[ -\frac{3\pi}{4}, -\frac{\pi}{2} \right] \cup \left[ -\frac{\pi}{2}, -\frac{\pi}{4} \right] \cup \left[ \frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[ \frac{\pi}{2}, \frac{3\pi}{4} \right]$
- 8.8 g) .....  $\left[ 0, \frac{3\pi}{4} \right] \cup \left[ \frac{7\pi}{4}, 2\pi \right]$
- 8.8 g) .....  $\left[ -\frac{\pi}{4}, \frac{3\pi}{4} \right]$
- 8.8 h) .....  $\left[ 0, \frac{3\pi}{8} \right] \cup \left[ \frac{7\pi}{8}, \frac{11\pi}{8} \right] \cup \left[ \frac{15\pi}{8}, 2\pi \right]$
- 8.8 h) .....  $\left[ -\pi, -\frac{5\pi}{8} \right] \cup \left[ -\frac{\pi}{8}, \frac{3\pi}{8} \right] \cup \left[ \frac{7\pi}{8}, \pi \right]$

## Fiche n° 9. Dérivation

### Réponses

9.1 a) .....  $6x^2 + 2x - 11$

9.1 b) .....  $5x^4 - 6x^2 + 4x - 15$

9.1 c) .....  $(2x^2 - 2x + 10) \exp(2x)$

9.1 d) .....  $(6x - 1) \ln(x - 2) + \frac{3x^2 - x}{x - 2}$

9.2 a) .....  $5(x^2 - 5x)^4(2x - 5)$

9.2 b) .....  $4(2x^3 + 4x - 1)(3x^2 + 2)$

9.2 c) .....  $8 \cos^2(x) - 6 \cos(x) \sin(x) - 4$

9.2 d) .....  $-3(3 \cos(x) - \sin(x))^2(3 \sin(x) + \cos(x))$

9.3 a) .....  $\frac{2x}{x^2 + 1}$

9.3 b) .....  $\frac{1}{x \ln(x)}$

9.3 c) .....  $(-2x^2 + 3x + 1) \exp(x^2 + x)$

9.3 d) .....  $6 \cos(2x) \exp(3 \sin(2x))$

9.4 a) .....  $\frac{6x}{(x^2 + 1)^2} \cos\left(\frac{2x^2 - 1}{x^2 + 1}\right)$

9.4 b) .....  $\frac{2x^2 + 2x - 8}{(x^2 + 4)^2} \sin\left(\frac{2x + 1}{x^2 + 4}\right)$

9.4 c) .....  $\frac{\cos(x)}{2\sqrt{\sin(x)}}$

9.4 d) .....  $\frac{\cos(\sqrt{x})}{2\sqrt{x}}$

9.5 a) .....  $\frac{(2x + 3)(2 \sin(x) + 3) - (x^2 + 3x) \times 2 \cos(x)}{(2 \sin(x) + 3)^2}$

9.5 b) .....  $\frac{2 - 3x}{2\sqrt{x}(3x + 2)^2}$

9.5 c) .....  $-\frac{2(x^2 + 1) \sin(2x + 1) + x \cos(2x + 1)}{(x^2 + 1)^2}$

9.5 d) .....  $\frac{(4x + 3) \ln(x) - 2x - 3}{(\ln(x))^2}$

9.6 a) .....  $2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$

9.6 b) .....  $\frac{9}{(9 - x^2)\sqrt{9 - x^2}}$

9.6 c) .....  $\frac{1}{1 - x^2}$

9.6 d) .....  $\frac{x \cos(x) - \sin(x)}{x \sin(x)}$

9.7 a) .....  $\frac{10x - 5}{(3 - x)^2(2 + x)^2}$

9.7 b) .....  $\frac{2}{x + 1} \left(x + \frac{1 + \sqrt{3}}{2}\right) \left(x + \frac{1 - \sqrt{3}}{2}\right)$

9.7 c) .....  $\frac{2x^2 + 2x + 5}{(x + 2)(x - 1)^2}$

9.7 d) .....  $\frac{x^2}{(x + 1)^2}$

9.7 e) .....  $\frac{2}{x(1 - \ln(x))^2}$

# Fiche n° 10. Primitives

## Réponses

10.1 a) .....	$\ln  t + 1 $	10.5 c) .....	$-\ln  \cos t $
10.1 b) .....	$-\frac{3}{t + 2}$	10.5 d) .....	$-\ln  1 - \sin t $
10.1 c) .....	$-\frac{3}{2(t + 2)^2}$	10.5 e) .....	$-2 \cos \sqrt{t}$
10.1 d) .....	$-\frac{\cos(4t)}{4}$	10.5 f) .....	$\frac{1}{\pi} \sin(\pi \ln t)$
10.2 a) .....	$\frac{2}{3}(1 + t)^{\frac{3}{2}} - \frac{3}{4}t^{\frac{4}{3}}$	10.5 g) .....	$\tan t - t$
10.2 b) .....	$\frac{1}{2}e^{2t+1}$	10.5 h) .....	$\frac{1}{2} \tan^2 t + \ln  \cos t $
10.2 c) .....	$\frac{1}{2} \text{Arcsin}(2t)$	10.5 i) .....	$\frac{1}{4} \tan^4 t$
10.2 d) .....	$\frac{1}{3} \text{Arctan}(3t)$	10.5 j) .....	$2\sqrt{\tan t}$
10.3 a) .....	$\frac{2}{3} \ln  1 + t^3 $	10.5 k) .....	$\frac{1}{\tan t}$
10.3 b) .....	$\frac{1}{6}(1 + 2t^2)^{\frac{3}{2}}$	10.5 l) .....	$\frac{1}{2} \frac{1}{(1 - \sin t)^2}$
10.3 c) .....	$-\sqrt{1 - t^2}$	10.5 m) .....	$\frac{1}{2} \text{Arctan}(2t)$
10.3 d) .....	$\frac{3}{4}(1 + 7t^2)^{\frac{2}{3}}$	10.5 n) .....	$\text{Arctan}(e^t)$
10.3 e) .....	$\frac{1}{6} \ln(1 + 3t^2)$	10.5 o) .....	$\frac{1}{2} (\text{Arcsin}(t))^2$
10.3 f) .....	$-\frac{1}{(1 + 3t^2)^2}$	10.5 p) .....	$\ln  \text{Arcsin}(t) $
10.4 a) .....	$\frac{1}{4} \ln^4 t$	10.6 a) .....	$\frac{t}{2} + \frac{\sin(2t)}{4}$
10.4 b) .....	$2\sqrt{\ln t}$	10.6 b) .....	$-\frac{\cos(4t)}{8} - \frac{\cos(2t)}{4}$
10.4 c) .....	$\frac{2}{(3 - e^{2t})^2}$	10.6 c) .....	$-\cos t + \frac{1}{3} \cos^3 t$
10.4 d) .....	$-\frac{2}{3t^{\frac{3}{2}}}$	10.6 d) .....	$\ln(1 + \sin^2 t)$
10.4 e) .....	$\ln  1 - e^{-t} + e^t $	10.6 e) .....	$\ln  \tan t $
10.4 f) .....	$-e^{\frac{1}{t}}$	10.6 f) .....	$-\cot t + \tan t$
10.5 a) .....	$-\frac{1}{3} \cos^3 t$	10.6 g) .....	$\frac{1}{4} \ln  \tan 2t $
10.5 b) .....	$e^{\sin t}$	10.7 a) .....	$t + \ln  t  - \frac{1}{t}$
		10.7 b) .....	$\ln  t  - \frac{1}{2t^2}$

10.7 c) .....	$t + \frac{t^3}{3} + \frac{t^5}{5}$	10.8 h) ..	$-\frac{3t^2 - 2t - 3}{(t^2 + 1)^2}$ puis $\frac{3}{2} \ln(t^2 + 1) - \text{Arctan}(t)$
10.7 d) .....	$t - \frac{t^2}{2} + \frac{t^3}{3}$	10.8 i) .....	$\cos t(3 \cos^2 t - 2)$ puis $-\frac{1}{3} \cos^3 t$
10.7 e) .....	$t - 2 \ln  t + 1 $	10.8 j) .....	$\sinh(t)^2 + \cosh^2(t)$ puis $\frac{1}{2} \sinh^2(t)$
10.7 f) .....	$t - \frac{t^2}{2} + \frac{t^3}{3} - \ln  t + 1 $	10.8 k) .....	$-\frac{2t \sin \frac{1}{t} + \cos \frac{1}{t}}{t^4}$ puis $\cos \frac{1}{t}$
10.7 g) .....	$\frac{1}{2} \ln(1 + t^2) - \text{Arctan}(t)$	10.8 l) .....	$\frac{2e^t}{(2 + e^t)^2}$ puis $\ln(2 + e^t)$
10.7 h) .....	$\ln  t + 1  + \frac{1}{t + 1}$	10.8 m) .....	$\frac{2 \cos t + 3}{(2 + 3 \cos t)^2}$ puis $-\frac{1}{3} \ln  2 + 3 \cos t $
10.8 a) .....	$2(t - 1)$ puis $\frac{1}{3}t^3 - t^2 + 5t$	10.8 n) .....	$\frac{1}{(1 - t^2)^{3/2}}$ puis $-\sqrt{1 - t^2}$
10.8 b) .....	$-\frac{1}{t^2} \left( \frac{2}{t} + 1 \right)$ puis $-\frac{1}{t} + \ln  t $	10.8 o) .....	$2 \frac{3 \cos^2 t - 1}{(1 + \cos^2 t)^2}$ puis $-\ln(1 + \cos^2(t))$
10.8 c) .....	$\frac{1}{2\sqrt{t}} + \frac{3}{t^4}$ puis $\frac{2}{3}t^{\frac{3}{2}} + \frac{1}{2t^2}$	10.8 p) .....	$(1 - 2t^2)e^{-t^2}$ puis $-\frac{1}{2}e^{-t^2}$
10.8 d) .....	$-\frac{4}{t^5} - \frac{3}{2} \frac{1}{t^{5/2}}$ puis $-\frac{1}{3} \frac{1}{t^3} - \frac{2}{\sqrt{t}}$	10.8 q) .....	$\frac{\ln t - 2}{t^2}$ puis $\ln t - \frac{1}{2} \ln^2 t$
10.8 e) .....	$2e^{2t} - 3e^{-3t}$ puis $\frac{1}{2}e^{2t} - \frac{1}{3}e^{-3t}$	10.8 r) .....	$\frac{1 + \ln t}{t^2 \ln^2 t}$ puis $\ln  \ln t $
10.8 f) .....	$3e^{3t-2}$ puis $\frac{1}{3}e^{3t-2}$	10.8 s) .....	$\frac{\cos \ln t - \sin \ln t}{t^2}$ puis $-\cos(\ln t)$
10.8 g) .....	$-\frac{t(t^3 + 2)}{(t - 1)^2(t^2 + t + 1)^2}$ puis $\frac{1}{3} \ln( t^3 - 1 )$	10.8 t) .....	$-\frac{e^t(e^{2t} - 1)}{(1 + e^{2t})^2}$ puis $\text{Arctan}(e^t)$



## Fiche n° 11. Calcul d'intégrales

### Réponses

11.1 a).....	Positif	11.3 e).....	$-\frac{1}{30}$	11.5 e).....	6	11.7 c).....	$e^2$
11.1 b).....	Négatif	11.3 f).....	$-\frac{2}{101}$	11.5 f).....	$\frac{1}{2} - \frac{\sqrt{3}}{2}$	11.7 d).....	$3e - 4$
11.1 c).....	Positif	11.4 a).....	0	11.6 a).....	0	11.7 e).....	$-\frac{1}{3}$
11.2 a).....	14	11.4 b).....	1	11.6 b).....	0	11.7 f).....	$\frac{5}{8}$
11.2 b).....	50	11.4 c).....	$\frac{1}{2}$	11.6 c).....	$\ln\left(\frac{2}{\sqrt{3}}\right)$	11.8 a).....	0
11.2 c).....	$\frac{147}{2}$	11.4 d).....	18	11.6 d).....	$-\frac{1}{384}$	11.8 b).....	$\frac{\pi}{4}$
11.2 d).....	-54	11.4 e).....	$e^2 - e^{-3}$	11.6 e).....	$\frac{1}{2}\left(1 - \frac{1}{e}\right)$	11.8 c).....	$\frac{99}{\ln 10}$
11.2 e).....	0	11.4 f).....	$-\ln 3$	11.6 f).....	$\frac{7}{48}$	11.8 d).....	$\frac{e - \frac{1}{e}}{2}$
11.2 f).....	$\frac{5}{2}$	11.5 a).....	78	11.7 a).....	$\frac{1}{2} - \frac{1}{e+1}$	11.8 e).....	$\frac{2}{3}$
11.3 a).....	8	11.5 b).....	$2(e^3 - 1)$	11.7 b).....	$\frac{17}{2}$	11.8 f).....	$\frac{2\pi}{9}$
11.3 b).....	-2	11.5 c).....	$\frac{1}{\pi} \ln\left(1 + \frac{\pi}{2}\right)$				
11.3 c).....	$\frac{8}{3}$	11.5 d).....	$\frac{\sqrt{2}}{6}$				
11.3 d).....	0						

## Fiche n° 12. Intégration par parties

### Réponses

12.1 a) .....  $\frac{\pi}{2} - 1$

12.1 b) .....  $\frac{5}{2}\text{ch}(2) - \frac{1}{2}\text{sh}(2) - \frac{3}{2}$

12.1 c) ..... 4

12.1 d) .....  $\frac{(\ln(2))^2 2^{\ln(2)} - 2 \ln(2) - 2^{\ln(2)} + 2}{(\ln(2))^2}$

12.1 e) ..... 1

12.1 f) .....  $2 \ln 2 - \frac{3}{4}$

12.1 g) .....  $\ln(2) - 2 + \frac{\pi}{2}$

12.1 h) .....  $\frac{\pi}{4} - \frac{1}{2}$

12.1 i) .....  $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

12.1 j) .....  $-\frac{2\sqrt{2}}{3} + \frac{4}{3}$

12.1 k) .....  $\frac{4}{3}\sqrt{2}\ln(2) - \frac{8}{9}\sqrt{2} + \frac{4}{9}$

12.1 l) .....  $\frac{\pi}{4} - \frac{1}{2}\ln 2 - \frac{\pi^2}{32}$

12.2 a) .....  $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto (-x+2)e^x \end{cases}$

12.2 b) .....  $\begin{cases} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto -\frac{1+\ln x}{x} \end{cases}$

12.2 c) .....  $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x \arctan(x) - \frac{1}{2} \ln(1+x^2) \end{cases}$

12.2 d) .....  $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x \text{sh}(x) - \text{ch}(x) \end{cases}$

12.3 a) .....  $\frac{5}{2} - e^2$

12.3 b) .....  $\frac{e^{\frac{\pi}{2}} + 1}{2}$

12.4 a) ...  $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto \frac{1}{2}(-\cos(x)\text{sh}(x) + \sin(x)\text{ch}(x)) \end{cases}$

12.4 b) .....  $\begin{cases} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto x \ln^2 x - 2x \ln x + 2x \end{cases}$

12.4 c) .....  $\begin{cases} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto x^3 \left( \frac{1}{3} \ln^2 x - \frac{2}{9} \ln x + \frac{2}{27} \right) \end{cases}$

12.4 d) ..  $\begin{cases} ]-1, 1[ \rightarrow \mathbb{R} \\ x \mapsto \frac{1}{2} e^{\arccos(x)} (x - \sqrt{1-x^2}) \end{cases}$

## Fiche n° 13. Changements de variable

### Réponses

13.1 a).....	$\frac{\pi}{2}$	13.2 e).....	$\frac{\pi}{12}$
13.1 b).....	$\frac{\pi}{6}$	13.2 f).....	$\frac{1}{2} \ln \frac{5}{2}$
13.1 c).....	$2 \arctan(e) - \frac{\pi}{2}$	13.3 a).....	$2e^2$
13.1 d).....	$\frac{1}{4}$	13.3 b).....	$-2((\sqrt{3}-1) \ln(\sqrt{3}-1) - 4 + 2\sqrt{3})$
13.1 e).....	$\frac{1}{12}$	13.4 a).....	$\left\{ \begin{array}{l} ]0, \frac{\pi}{2}[ \rightarrow \mathbb{R} \\ x \mapsto \tan x + \ln \tan(x) \end{array} \right.$
13.1 f).....	$2 \ln\left(\frac{3}{2}\right)$	13.4 b).....	$\left\{ \begin{array}{l} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto \frac{x}{2} - \frac{e^{-2x}}{4} \end{array} \right.$
13.2 a).....	$\frac{\pi}{3\sqrt{3}}$	13.4 c).....	$\left\{ \begin{array}{l} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto 2 \arctan(\sqrt{e^x - 1}) \end{array} \right.$
13.2 b).....	$\frac{1}{2} \ln\left(\frac{2e+1}{3}\right)$	13.4 d).....	$\left\{ \begin{array}{l} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto \frac{3}{2} \ln(x^{\frac{2}{3}} + 1) \end{array} \right.$
13.2 c).....	$\frac{\pi}{2}$	13.4 e).....	$\left\{ \begin{array}{l} ]1, +\infty[ \rightarrow \mathbb{R} \\ x \mapsto \arctan \sqrt{x^2 - 1} \end{array} \right.$
13.2 d).....	$\frac{1}{4} + \frac{\pi}{8}$		

# Fiche n° 14. Intégration des fractions rationnelles

## Réponses

14.1 a) .....  $\ln\left(\frac{3}{2}\right)$

14.1 b) .....  $\frac{1}{2}\ln\left(\frac{5}{3}\right)$

14.2 a) .....  $2\ln\frac{9}{10}$

14.2 b) .....  $\ln(a+1)$

14.3 a) .....  $\frac{3}{2} + \ln(3) - \ln(2)$

14.3 b) .....  $-\frac{1}{48} + \frac{51}{64}\ln\frac{21}{19}$

14.4 a) .....  $\ln\left(\frac{7}{3}\right)$

14.4 b) .....  $\ln\frac{33}{28}$

14.5 a) .....  $\ln\left(2\sqrt{\sqrt{2}-1}\right)$

14.5 b) .....  $\frac{1}{2a}\ln\left(\frac{a+1}{2}\right)$

14.6 a) ..... 1 et 2

14.6 b) .....  $A = -1$  et  $B = 1$

14.6 c) .....  $2\ln\frac{4}{3}$

14.7 a) .....  $\ln\frac{1}{3}$

14.7 b) .....  $2\ln\frac{4}{3}$

14.7 c) .....  $\frac{1}{2}\ln\frac{3}{2}$

14.7 d) .....  $\frac{1}{4}\ln\frac{1}{5}$

14.8 .....  $\frac{1}{2\sqrt{a}}\ln\left(\frac{\sqrt{a}-a}{a+\sqrt{a}}\right)$

14.9 a) .....  $\frac{1}{a^2+x^2}$

14.9 b) .....  $\frac{1}{a}\arctan\left(\frac{x}{a}\right)$

14.10 a) .....  $\frac{\pi}{4}$

14.10 b) .....  $\frac{\pi}{6\sqrt{3}}$

14.11 .....  $\frac{\pi}{2\sqrt{2}}$

14.12 a) .....  $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

14.12 b) .....  $2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$

14.12 c) ..  $\sqrt{2}\left(x + \frac{1}{4}\right)^2 + \sqrt{2}\frac{15}{16}$

14.12 d) .....  $a\left(x + \frac{a}{2}\right)^2 + \frac{3a^3}{4}$

14.13 a) .....  $\frac{1}{2}$

14.13 b) .....  $\frac{2\pi}{3\sqrt{3}}$

14.13 c) .....  $\frac{2\pi}{3\sqrt{3}}$

14.13 d) .....  $\ln(2)$

14.14 a) .....  $\frac{\pi}{12}$

14.14 b) .....  $\ln\left(\frac{a^2}{a^2-1}\right)$

14.15 .....  $\frac{1}{3}\left(\ln(2) + \frac{\pi}{\sqrt{3}}\right)$

## Fiche n° 15. Systèmes linéaires

### Réponses

15.1 a) .....  $\{(3, 1)\}$

15.1 b) .....  $\{(7, 2)\}$

15.1 c) .....  $\left\{\left(\frac{1}{3}, \frac{2}{3}\right)\right\}$

15.1 d) .....  $\left\{\left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{2}\right)\right\}$

15.2 a) .....  $\left\{\left(1 - \frac{a}{4}, \frac{-1}{2} + \frac{3}{8}a\right)\right\}$

15.2 b) .....  $(2, -3)$

15.2 c) .....  $\left\{\left(\frac{1}{13}a + \frac{5}{13}a^2, \frac{2}{13}a - \frac{3}{13}a^2\right)\right\}$

15.2 d) .....  $(a - 2a^2, a + a^2)$

15.3 a) .....  $\{(1 + z, -z, z); z \in \mathbb{R}\}$

15.3 b) .....  $\{(1, y, 3 + 2y); y \in \mathbb{R}\}$

15.3 c) .....  $\left\{\left(\frac{13}{6} - \frac{5}{3}z, -\frac{1}{3} + \frac{4}{3}z, z\right); z \in \mathbb{R}\right\}$

15.3 d) .....  $\left\{\left(x, \frac{-5}{12} - \frac{3}{2}x, \frac{-25}{24} - \frac{7}{4}x\right); x \in \mathbb{R}\right\}$

15.4 a) .....  $\{(2, -1, 3)\}$

15.4 b) .....  $\{(-1, 4, 2)\}$

15.4 c) .....  $\emptyset$

15.4 d) .....  $\left\{\left(-\frac{2}{7} - z, \frac{3}{7}, z\right); z \in \mathbb{R}\right\}$

15.5 a) .....  $\{(1, 1/2, 1/2)\}$

15.5 b) .....  $\emptyset$

15.5 c) .....  $\{(5z, 1 - 4z, z); z \in \mathbb{R}\}$

15.5 d) .....  $\left\{\left(1, \frac{1}{a+2}, \frac{1}{a+2}\right)\right\}$

15.6 a) .....  $\{(5, 3, -1)\}$

15.6 b) .....  $\emptyset$

15.6 c) .....  $\left\{\left(\frac{a^2 + a - 1}{a^3 - 1}c, \frac{a^2 - a + 1}{a^3 - 1}c, \frac{-a^2 + a + 1}{a^3 - 1}c\right)\right\}$

15.7 a) .....  $\{(0, 0, 0)\}$

15.7 b) .....  $\{(x, y, -x - y); (x, y) \in \mathbb{R}^2\}$

15.7 c) .....  $\{(x, x, x); x \in \mathbb{R}\}$

## Fiche n° 16. Nombres complexes

### Réponses

16.1 a) .....  $4 + 32i$

16.1 b) .....  $13 - i$

16.1 c) .....  $7 - 24i$

16.1 d) .....  $5$

16.1 e) ..  $-119 + 120i$

16.1 f) .....  $\frac{3}{10} + \frac{1}{10}i$

16.1 g) .....  $\frac{4}{29} - \frac{19}{29}i$

16.1 h) .....  $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

16.2 a) .....  $12$

16.2 b) .....  $8e^{i\pi}$

16.2 c) .....  $\sqrt{3}e^{i\frac{\pi}{2}}$

16.2 d) .....  $2e^{-i\frac{\pi}{2}}$

16.2 e) .....  $2e^{i\frac{8\pi}{5}}$

16.2 f) .....  $5\sqrt{2}e^{-i\frac{\pi}{4}}$

16.2 g) .....  $10e^{i\frac{2\pi}{3}}$

16.2 h)  $2 \cos\left(\frac{\pi}{12}\right) e^{i\frac{\pi}{4}}$

16.3 a) .....  $1$

16.3 b) ...  $\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

16.3 c) ..  $-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

# Fiche n° 17. Trigonométrie et nombres complexes

## Réponses

17.1 a) .....  $\frac{1}{4} \cos(3x) + \frac{3}{4} \cos(x)$

17.1 b) .....  $-\frac{1}{4} \cos(4x) + \frac{1}{2} \cos(2x) - \frac{1}{4}$

17.1 c) ...  $-\frac{1}{8} \cos(6x) + \frac{1}{4} \cos(4x) - \frac{3}{8} \cos(2x) + \frac{1}{4}$

17.1 d) ...  $-\frac{\sin(9x)}{8} + \frac{3 \sin(5x)}{8} - \frac{\sin(3x)}{8} - \frac{3 \sin(x)}{8}$

17.1 e) ....  $\frac{\cos(9x)}{8} + \frac{3 \cos(5x)}{8} + \frac{\cos(3x)}{8} + \frac{3 \cos(x)}{8}$

17.1 f) .....  $-\frac{1}{4} \sin(11x) + \frac{1}{4} \sin(5x) + \frac{1}{2} \sin(3x)$

17.2 a) .....  $2 \cos\left(\frac{\pi}{12}\right) e^{i \frac{\pi}{12}}$

17.2 b) .....  $\left(-2 \cos\left(\frac{7\pi}{12}\right)\right) e^{-i \frac{5\pi}{12}}$

17.2 c) .....  $2 \sin\left(\frac{\pi}{12}\right) e^{-\frac{7i\pi}{12}}$

17.2 d) .....  $2 \cos\left(\frac{5\pi}{12}\right) e^{\frac{5i\pi}{12}}$

17.2 e) .....  $2 \cos\left(\frac{\pi}{12}\right) e^{i \frac{13\pi}{12}}$

17.2 f) .....  $2 \sin\left(\frac{\pi}{24}\right) e^{-i \frac{11\pi}{24}}$

17.2 g) .....  $\frac{\cos\left(\frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{24}\right)} e^{\frac{13i\pi}{24}}$

17.2 h) .....  $2^{27} \cos^{27}\left(\frac{\pi}{12}\right) e^{i \frac{\pi}{4}}$

17.3 a) .....  $2 \cos\left(\frac{\pi}{12}\right) e^{i \frac{5\pi}{12}}$

17.3 b) .....  $2 \sin\left(\frac{\pi}{12}\right) e^{-i \frac{\pi}{12}}$

17.4 a) .....  $4 \cos^3(x) - 3 \cos(x)$

17.4 b) .....  $4 \cos^3(x) \sin(x) - 4 \cos(x) \sin^3(x)$

17.5 a) .....  $2 \cos(2x) \cos(x)$

17.5 b) .....  $2 \cos(4x) \sin(x)$

17.5 c) .....  $2 \sin(x) \sin(2x)$

17.5 d) .....  $2 \sin(4x) \cos(x)$

17.6 a) .....  $\frac{\sin\left(\frac{3x}{2}\right) \sin(2x)}{\sin\left(\frac{x}{2}\right)}$

17.6 b) .....  $\frac{\sin(8x)}{2 \sin(x)}$

17.6 c) .....  $0$

17.7 a) .....  $\frac{e^\pi + 1}{2}$

17.7 b) .....  $\frac{1}{5}(e^\pi - 2)$

# Fiche n° 18. Sommes et produits

## Réponses

18.1 a) .....  $\boxed{n(n+2)}$

18.1 b) .....  $\boxed{\frac{7(n+1)(n+4)}{2}}$

18.1 c) .....  $\boxed{\frac{n(5n+1)}{2}}$

18.1 d) .....  $\boxed{\frac{(n-2)(n-7)}{6}}$

18.2 a) .....  $\boxed{\frac{n(n+1)(n+2)}{3}}$

18.2 b) ...  $\boxed{n(n+1)(n^2+n+4)}$

18.2 c) .....  $\boxed{\frac{9}{2}(3^{n-2}-1)}$

18.2 d) .....  $\boxed{\frac{5^{n+1}-2^{n+1}}{3}}$

18.2 e) ...  $\boxed{\frac{7}{6}(7^n-1)+n(n+4)}$

18.2 f) .....  $\boxed{\frac{n+1}{2n}}$

18.3 a) .....  $\boxed{2^{q-p+1}}$

18.3 b) .....  $\boxed{3^{\frac{n(n+1)}{2}}}$

18.3 c) .....  $\boxed{5^n(n!)^{\frac{3}{2}}}$

18.3 d) .....  $\boxed{0}$

18.4 a) .....  $\boxed{\frac{n(n+1)}{2}}$

18.4 b) .....  $\boxed{0}$

18.4 c) .....  $\boxed{n2^{n+1}+2(1-2^n)}$

18.4 d) .....  $\boxed{\frac{n^2(n+1)^2}{4}}$

18.5 a) .....  $\boxed{(n+3)^3-2^3}$

18.5 b) .....  $\boxed{\ln(n+1)}$

18.5 c) .....  $\boxed{1-\frac{1}{(n+1)!}}$

18.5 d) .....  $\boxed{(n+1)!-1}$

18.6 a) .....  $\boxed{n+1}$

18.6 b) .....  $\boxed{1-4n^2}$

18.6 c) .....  $\boxed{\frac{1}{n}}$

18.6 d) .....  $\boxed{\frac{n+1}{2n}}$

18.7 a) .....  $\boxed{1-\frac{1}{n+1}}$

18.7 b) .....  $\boxed{\frac{1}{2}-\frac{1}{n+3}}$

18.8 a) .....  $\boxed{2n^2+n}$

18.8 b) .....  $\boxed{\frac{n(3n+1)}{2}}$

18.9 a) .....  $\boxed{\frac{n^2(n+1)}{2}}$

18.9 b) .....  $\boxed{\frac{n(n+3)}{4}}$

18.9 c) .....  $\boxed{\frac{n(n^2-1)}{2}}$

18.9 d) ..  $\boxed{\frac{n(n+1)(7n^2+13n+4)}{12}}$

18.9 e) .....  $\boxed{\frac{n(n+1)}{2}\ln(n!)}$

18.9 f) .....  $\boxed{\frac{n(n+1)(4n-1)}{6}}$



# Fiche n° 19. Coefficients binomiaux

## Réponses

- 19.1 a)** .....  $10\ 100$   
**19.1 b)** .....  $720$   
**19.1 c)** .....  $\frac{1}{30}$   
**19.1 d)** .....  $15$   
**19.1 e)** .....  $56$   
**19.1 f)** .....  $140$   
**19.2 a)** .....  $\frac{9!}{5!}$   
**19.2 b)** .....  $\binom{9}{4}$   
**19.2 c)** .....  $2^n \times n!$   
**19.2 d)** .....  $\frac{(2n+1)!}{2^n \times n!}$   
**19.3 a)** .....  $\frac{n(n-1)}{2}$   
**19.3 b)** .....  $\frac{n(n-1)(n-2)}{6}$   
**19.3 c)** .....  $\frac{k+1}{n-k}$   
**19.3 d)** .....  $(n+2)(n+1)$   
**19.3 e)** .....  $\frac{1}{(n+1)!}$   
**19.3 f)** .....  $\frac{n! \times (n-3)}{2^{2n+2}}$   
**19.4 a)** .....  $\frac{(n+1)^3}{n \times (n+2)!}$   
**19.4 b)** .....  $\frac{3(3n+2)(3n+1)}{a^3(n+1)^2}$   
**19.5 a)** .....  $3^n$   
**19.5 b)** .....  $0$   
**19.5 c)** .....  $6^n$   
**19.5 d)** .....  $12 \times 15^n$   
**19.6 a)** .....  $2 \times \sum_{p=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2p}$   
**19.6 b)** .....  $2^{n-1}$   
**19.7 a)** .....  $2^n$   
**19.7 b)** .....  $n2^{n-1}$   
**19.7 c)** .....  $n(n+1)2^{n-2}$   
**19.7 d)** .....  $\frac{2^{n+1} - 1}{n+1}$   
**19.8 a)** .....  $\binom{2n}{n}$   
**19.8 b)** .....  $\sum_{k=0}^n \binom{n}{k}^2$   
**19.8 c)** .....  $\binom{2n}{n}$

# Fiche n° 20. Manipulation des fonctions usuelles

## Réponses

- 20.1 a)** .....  $\frac{\pi}{6}$       **20.4 c)** .....  $\frac{\ln(3)}{\ln(2)}$       **20.7 d)** .  $[-\ln(4+\sqrt{15}), \ln(4+\sqrt{15})]$   
**20.1 b)** .....  $2$       **20.4 d)** .....  $\frac{\ln(4)}{\ln(20/3)}$       **20.7 e)** .....  $[\ln(3 + \sqrt{10}), +\infty[$   
**20.1 c)** .....  $\frac{\pi}{4}$       **20.5 a)** .....  $\frac{\ln\left(\frac{\sqrt{17}-1}{2}\right)}{\ln(2)}$       **20.7 f)** .....  $]-\infty, \frac{1}{2} \ln(3)]$   
**20.1 d)** .....  $\frac{\pi}{6}$       **20.5 b)** .....  $\left\{0; \frac{1}{2}\right\}$       **20.8 a)** ...  $x \mapsto \ln(2) \times 2^x + 2x$   
**20.1 e)** .....  $\frac{\pi}{4}$       **20.5 c)** .....  $1 - \frac{\ln(2)}{\ln(3)}$       **20.8 b)** .  $x \mapsto \frac{15^x \ln(3/5) + 3^x \ln(3)}{(5^x + 1)^2}$   
**20.1 f)** .....  $\frac{\pi}{3}$       **20.5 d)** .....  $\frac{\ln\left(\frac{\sqrt{5}-1}{2}\right)}{\ln(3)}$       **20.8 c)** .....  $x \mapsto (\ln(x) + 1)x^x$   
**20.2 a)** .....  $1$       **20.6 a)** .....  $1$       **20.8 d)** .  $x \mapsto \frac{\pi}{2\sqrt{1-x^2} \arccos(x)^2}$   
**20.2 b)** .....  $0$       **20.6 b)** .....  $0$       **20.9 a)** .....  $x \mapsto 2x \frac{1}{\sqrt{1-x^4}}$   
**20.2 c)** .....  $\frac{5}{4}$       **20.6 c)** .....  $\{2k\pi, k \in \mathbb{Z}\}$       **20.9 b)** ...  $x \mapsto \operatorname{ch}^2(x) + \operatorname{sh}^2(x)$   
**20.2 d)** .....  $\frac{4}{3}$       **20.6 d)** .  $\left\{\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}$   
 $\cup \left\{\frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}$       **20.9 c)** .....  $x \mapsto \frac{1 - \operatorname{th}^2(x)}{1 + \operatorname{th}^2(x)}$   
**20.2 e)** .....  $\frac{13}{12}$       **20.6 e)** .  $\left\{\frac{1}{3} + 2k\pi, k \in \mathbb{Z}\right\}$   
 $\cup \left\{\pi - \frac{1}{3} + 2k\pi, k \in \mathbb{Z}\right\}$       **20.9 d)** ....  $x \mapsto \operatorname{sh}(x) \operatorname{ch}(\operatorname{ch}(x))$   
**20.2 f)** .....  $\frac{3}{5}$       **20.6 f)** .....  $1$       **20.10 a)** .....  $x \mapsto 0$   
**20.3 a)** .....  $\operatorname{sh}(x + y)$       **20.7 a)** .  $\{\ln(\sqrt{5} - 2); \ln(\sqrt{5} + 2)\}$       **20.10 b)** .....  $x \mapsto 0$   
**20.3 b)** .....  $\operatorname{ch}(x - y)$       **20.7 b)** .....  $\ln(1 + \sqrt{2})$       **20.11 a)** .  $x \mapsto (\ln(x) + 1)x^x e^{-x^2x}$   
**20.4 a)** .....  $\frac{\ln(2)}{\ln(3)}$       **20.7 c)** .....  $\frac{1}{2} \ln(2)$       **20.11 b)** .  $x \mapsto \frac{\operatorname{sh}(x)}{\operatorname{ch}(x)^2} \frac{1}{2\sqrt{\ln(\operatorname{ch}(x))}}$   
**20.4 b)** .....  $1$       **20.11 c)** .....  $x \mapsto \arcsin(x)$   
**20.11 d)** .....  $x \mapsto \arctan(x)$

## Fiche n° 21. Suites numériques

### Réponses

21.1 a).....	$\frac{12}{5}$	21.6 a).....	21	21.9 a).....	$\frac{\pi\sqrt{5}}{5}$
21.1 b).....	8	21.6 b).....	10 000	21.9 b).....	$\frac{11\sqrt{5}}{25}$
21.1 c).....	$\frac{(2n+5) \cdot 2^{n+3}}{5}$	21.6 c).....	2 001	21.10 a).....	$3^n + (-2)^n$
21.1 d).....	$\frac{3(2n+1) \cdot 2^{3n+2}}{5}$	21.6 d).....	10 201	21.10 b).....	211
21.2 a).....	13	21.7 a).....	$\frac{17}{24}$	21.11 a) ..	$\frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2}$
21.2 b).....	29	21.7 b).....	$\frac{1}{24}$	21.11 b).....	$2\sqrt{2}$
21.3 a).....	$2^{\frac{1}{8}}$	21.8 a).....	$\frac{3}{512}$	21.12 a).....	257
21.3 b).....	$2^{\frac{1}{64}}$	21.8 b).....	$\frac{3069}{512}$	21.12 b).....	65 537
21.4 a).....	2	21.8 c).....	$\frac{3}{1\ 024}$	21.12 c).....	$F_n$
21.4 b).....	2	21.8 d).....	$\frac{6141}{1024}$	21.12 d).....	$F_{n+1} - 2$
21.5 a).....	$2n \ln(n)$			21.12 e).....	$F_{n+1} + 2^{2^n+1}$
21.5 b).....	$4n \ln(2n)$			21.12 f).....	$F_{n+2}$

## Fiche n° 22. Développements limités

### Réponses

22.1 a) .....  $3x - x^2 + \frac{x^3}{2} - \frac{x^4}{2} + o_{x \rightarrow 0}(x^4)$

22.1 b) .....  $x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + o_{x \rightarrow 0}(x^4)$

22.1 c) .....  $\frac{x^3}{2} - \frac{x^5}{24} + o_{x \rightarrow 0}(x^6)$

22.1 d) .....  $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + o_{x \rightarrow 0}(x^6)$

22.2 a) .....  $e - \frac{ex}{2} + \frac{11ex^2}{24} - \frac{7ex^3}{16} + \frac{2447ex^4}{5760} + o_{x \rightarrow 0}(x^5)$

22.2 b) .....  $1 - \frac{1}{4}x^2 - \frac{1}{96}x^4 - \frac{19}{5760}x^6 + o_{x \rightarrow 0}(x^7)$

22.2 c) .....  $e \left( 1 + ix - x^2 - \frac{5}{6}ix^3 \right) + o_{x \rightarrow 0}(x^3)$

22.2 d) .....  $1 - x + \frac{3}{2}(x-1)^2 + o_{x \rightarrow 1}((x-1)^2)$

22.3 a) .....  $1 - \frac{3\pi^2}{8} \left( x - \frac{\pi}{3} \right)^2 + o_{x \rightarrow \frac{\pi}{3}} \left( \left( x - \frac{\pi}{3} \right)^2 \right)$

22.3 b) .....  $1 + 2 \left( x - \frac{\pi}{4} \right) + 2 \left( x - \frac{\pi}{4} \right)^2 + \frac{8}{3} \left( x - \frac{\pi}{4} \right)^3 + o_{x \rightarrow \frac{\pi}{4}} \left( \left( x - \frac{\pi}{4} \right)^4 \right)$

22.3 c) .....  $-1 + \frac{\pi^2}{8} \left( x - \frac{\pi}{2} \right)^4 - \frac{\pi^2}{48} \left( x - \frac{\pi}{2} \right)^6 + o_{x \rightarrow \frac{\pi}{2}} \left( \left( x - \frac{\pi}{2} \right)^7 \right)$

22.4 a) .....  $-\frac{1}{2x} + \frac{1}{12} - \frac{1}{720}x^2 + o_{x \rightarrow 0}(x^2)$

22.4 b) .....  $\frac{1}{x^2} - \frac{1}{x^3} + \frac{5}{6x^4} - \frac{5}{6x^5} + o_{x \rightarrow +\infty} \left( \frac{1}{x^6} \right)$

22.4 c) .....  $-\ln(x) + 1 - \frac{1}{2x} + \frac{1}{3x^2} - \frac{1}{4x^3} + o_{x \rightarrow +\infty} \left( \frac{1}{x^3} \right)$

22.4 d) .....  $e^{-\frac{1}{2}} \left( e^x + \frac{e^x}{3x} - \frac{7e^x}{36x^2} \right) + o_{x \rightarrow +\infty} \left( \frac{e^x}{x^2} \right)$

## Fiche n° 23. Arithmétique

### Réponses

23.1 a).....	$(6, 7)$	23.4 .....	1	23.7 a) .....	$(-5, 2)$	23.9 d).	il est premier
23.1 b).....	$(-7, 2)$	23.5 a) .....	154	23.7 b) ..	$8 \pmod{13}$	23.10 a) .....	67
23.1 c).....	$(-6, 7)$	23.5 b).....	$\frac{65}{18}$	23.7 c) ..	$11 \pmod{13}$	23.10 b) .....	7
23.1 d).....	$(7, 2)$	23.5 c).....	29 160	23.8 a).....	5	23.11 a) .....	1
23.2 a).....	20	23.5 d).....	$\frac{1}{29\ 160}$	23.8 b) ..	$(2023, 6406)$	23.11 b) .....	1
23.2 b).....	4	23.6 a) ....	$(216, 192)$	23.9 a) ...	$2 \times 3 \times 337$	23.11 c).....	6
23.3 a).....	2	23.6 b) .....	$(12, 30)$	23.9 b).....	$7 \times 17^2$	23.11 d) .....	5
23.3 b).....	4			23.9 c).....	$43 \times 47$	23.11 e) .....	66
						23.11 f).....	2

## Fiche n° 24. Polynômes

### Réponses

- 24.1 a) .....  $\begin{cases} Q = X^2 + 2X + 1 \\ R = 2 \end{cases}$
- 24.1 b) .....  $\begin{cases} Q = X^2 - 4X + 7 \\ R = -3X - 8 \end{cases}$
- 24.1 c) .....  $\begin{cases} Q = X^2 - 1 \\ R = -X^2 + X + 1 \end{cases}$
- 24.1 d) .....  $\begin{cases} Q = 13X + \frac{25}{2} \\ R = \frac{1}{2}(29X^2 - 5X - 23) \end{cases}$
- 24.2 a) .....  $R = 1$
- 24.2 b) .....  $R = 0$
- 24.2 c) .....  $R = -2nX + 4n - 1$
- 24.2 d) .....  $R = X^2 + X - 1$
- 24.3 a) .....  $R = 2X - 3$
- 24.3 b) .....  $R = -2X^3 - 3X^2 + 1$
- 24.3 c) .....  $R = -8X^3 + 21X^2 - 20X + 5$
- 24.3 d) .....  $R = -29X^3 + 11X^2 + 2X - 1$
- 24.4 a) .....  $R = -36X + 24$
- 24.4 b) .....  $24 - 36i$
- 24.5 a) .....  $R = -108X - 150$
- 24.5 b) .....  $-150 - 108\sqrt{2}$
- 24.6 a) .....  $76 - 92\sqrt{2}$
- 24.6 b) .....  $8 - 206i$
- 24.7 a) .....  $(X - 1)^2(X^2 + 1)$
- 24.7 b) .....  $(X^2 - 2X + 2)(X^2 - 2X + 5)$
- 24.7 c) .....  $(X - 1)^2(X^2 + 1)(X + 1)(X - 2)$

# Fiche n° 25. Décomposition en éléments simples

## Réponses

25.1 a).....  $X - 3 - \frac{1}{X} + \frac{1}{X+1} + \frac{7}{X+2}$

25.1 b).....  $1 - \frac{2}{X} + \frac{1}{2(X+1)} + \frac{3}{2(X-1)}$

25.1 c).....  $1 + \frac{\pi}{2(X-\pi)} - \frac{\pi}{2(X+\pi)}$

25.2 a).....  $\frac{e-1}{(e-2)(X+e)} + \frac{1}{(2-e)(X+2)}$

25.2 b).....  $\frac{3}{2(X-1)} - \frac{1+i}{4(X-i)} - \frac{1-i}{4(X+i)}$

25.2 c).....  $1 - \frac{5}{(\sqrt{2}+\sqrt{3})(X+\sqrt{3})} - \frac{4}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-X)}$

25.3 a).....  $\frac{-3}{X-2} + \frac{1}{X-3} + \frac{2}{X-1} + \frac{1}{(X-1)^2}$

25.3 b).....  $\frac{2}{X} + \frac{2}{X^2} - \frac{11}{4(X-1)} + \frac{3}{2(X-1)^2} + \frac{3}{4(X+1)}$

25.3 c).....  $\frac{1}{\pi^2 X} - \frac{1}{\pi^2(X+\pi)} - \frac{1+\pi}{\pi(X+\pi)^2}$

25.3 d).....  $\frac{2}{X-i} + \frac{1}{(X-i)^2} - \frac{2}{X-(1+i)} + \frac{1}{(X-(1+i))^2}$

25.4 a).....  $\frac{1}{X+1} - \frac{1}{2(X-1)} - \frac{1+3i}{4(X-i)} - \frac{1-3i}{4(X+i)}$

25.4 b).....  $\frac{1}{2X} + \frac{5}{6(X+2)} + \frac{2}{3(X-1)} + \frac{1}{(X-1)^2}$

25.5 a).....  $\frac{1}{2(n+1)} - \frac{1}{2n} + \frac{1}{4}$

25.5 b).....  $-\frac{2}{n+2} + \frac{1}{n} - \frac{1}{3}$

25.6 a).....  $\frac{2}{X+1} + \frac{1}{(X+1)^2} + \frac{1-2X}{X^2+1}$

25.6 b).....  $\frac{1}{2(X-1)} - \frac{3}{2(X+1)} + \frac{X-1}{X^2+X+1}$

25.7 a).....  $1 - 2\ln(3)$

25.7 b).....  $-\frac{1}{2}\ln(3) + \frac{2}{3}\ln(2)$

25.7 c).....  $\frac{2}{3} - 4\ln(2) + 2\ln(3)$

25.7 d).....  $\frac{1}{18} - \frac{1}{9}\ln(5) + \frac{2}{9}\ln(2)$

25.7 e).....  $\frac{\pi}{8}$

25.7 f).....  $\frac{1}{2}\ln(2) - \frac{1}{4}\ln(3)$

25.8 a).....  $\frac{1}{2}\ln\left|\frac{x-1}{1+x}\right|$

25.8 b).....  $x \mapsto \frac{1}{4(1-2x)^2}$

25.8 c).....  $\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right)$

25.8 d).....  $\frac{\sqrt{3}}{2}\arctan\left(\frac{2}{\sqrt{3}}X + \frac{1}{\sqrt{3}}\right)$

25.8 e).....  $2$

25.8 f).....  $\frac{x^2}{2} + 2x + \frac{1}{6}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{16}{3}\ln|x-2|$

25.8 g).....  $x \mapsto \frac{1}{6}\ln(x^2+2) - \frac{1}{3}\ln|x+1| + \frac{\sqrt{2}}{3}\arctan\left(\frac{x}{\sqrt{2}}\right)$

25.8 h).....  $x \mapsto \frac{1}{2}\frac{2x-1}{x^2-1} + \frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|$

# Fiche n° 26. Calcul matriciel

## Réponses

26.1 a) .....  $\begin{pmatrix} 1 & -3 & -1 \\ 3 & 3 & 4 \\ 9 & -7 & 3 \end{pmatrix}$

26.1 b) .....  $\begin{pmatrix} -2 & -6 & -5 \\ 15 & -1 & 11 \\ 18 & -26 & -1 \end{pmatrix}$

26.1 c) ..... 17 (matrice  $1 \times 1$ )

26.1 d) .....  $\begin{pmatrix} 1 & 7 & -2 \\ 2 & 14 & -4 \\ -1 & -7 & 2 \end{pmatrix}$

26.1 e) .....  $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$

26.1 f) .....  $(-5 \ 15 \ 3)$

26.1 g) .....  $\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

26.1 h) .....  $\begin{pmatrix} 5 & 3 & -1 & 1 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

26.1 i) .....  $\begin{pmatrix} 1 & 7 & -2 \\ 7 & 49 & -14 \\ -2 & -14 & 4 \end{pmatrix}$

26.2 a) .....  $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

26.2 b) .....  $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

26.2 c) .....  $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

26.2 d) .....  $\begin{pmatrix} 4 & 5 \\ 0 & 9 \end{pmatrix}$

26.2 e) .....  $\begin{pmatrix} 8 & 19 \\ 0 & 27 \end{pmatrix}$

26.2 f) .....  $\begin{pmatrix} 2^k & 3^k - 2^k \\ 0 & 3^k \end{pmatrix}$

26.2 g) .....  $\begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$

26.2 h) .....  $\begin{pmatrix} \cos(3\theta) & -\sin(3\theta) \\ \sin(3\theta) & \cos(3\theta) \end{pmatrix}$

26.2 i) .....  $\begin{pmatrix} \cos(k\theta) & -\sin(k\theta) \\ \sin(k\theta) & \cos(k\theta) \end{pmatrix}$

26.2 j) .....  $\begin{pmatrix} n & \cdots & n \\ \vdots & (n) & \vdots \\ n & \cdots & n \end{pmatrix}$

26.2 k) .....  $\begin{pmatrix} n^2 & \cdots & n^2 \\ \vdots & (n^2) & \vdots \\ n^2 & \cdots & n^2 \end{pmatrix}$

26.2 l) .....  $n^{k-1}D$

26.3 a) .....  $2 \times 3^{j-i} \times 5^{i-1}$

26.3 b) .....  $2^{i+1}3^{j-i}(2^n - 1)$

26.3 c) .....  $2 \times 3^{i+j} \left(1 - \left(\frac{2}{3}\right)^n\right)$

26.3 d) .....  $\begin{pmatrix} i-1 \\ j \end{pmatrix} + \begin{pmatrix} i-1 \\ j-2 \end{pmatrix}$

26.4 a) .....  $2^{i-j} \begin{pmatrix} i-1 \\ j-1 \end{pmatrix}$

26.4 b) .....  $(1 - \delta_{i,1})(\delta_{i-1,j+1} + \delta_{i,j}) + (1 - \delta_{i,n})(\delta_{i,j} + \delta_{i+1,j-1})$

26.5 a) .....  $\frac{1}{2(\pi - e)} \begin{pmatrix} 2 & -e \\ -2 & \pi \end{pmatrix}$

26.5 b) .....  $\frac{1}{3} \begin{pmatrix} 1 & -1 - 2i \\ 1 & -1 + i \end{pmatrix}$

26.5 c) .....  $\frac{1}{2} \begin{pmatrix} 5 & 2 & -1 \\ 3 & 2 & -1 \\ -6 & -2 & 2 \end{pmatrix}$

26.5 d) .....  $\frac{1}{4\pi} \begin{pmatrix} 0 & 4 & 0 \\ 0 & -2 & -2 \\ 2 & -1 & 1 \end{pmatrix}$

26.5 e) .....  $\frac{1}{8} \begin{pmatrix} 8 & 4 & -2 \\ -16 & -6 & 7 \\ 0 & -2 & 1 \end{pmatrix}$

26.5 f) .....  $\frac{1}{6} \begin{pmatrix} -2 & 2 & 2 \\ 1 & -1 & 2 \\ 4 & 2 & -4 \end{pmatrix}$



**26.5 g)** .....  $\frac{1}{2} \begin{pmatrix} 4 & -2 & 2 & 0 \\ 8 & -6 & 4 & 2 \\ -7 & 5 & -3 & -1 \\ -5 & 3 & -1 & -1 \end{pmatrix}$

**26.5 h)** ..... Non inversible!

**26.5 i)** .....  $\frac{1}{2} \begin{pmatrix} 0 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

**26.6 a)** .....  $\lambda \neq 1$

**26.6 b)** .....  $\frac{1}{1-\lambda} \begin{pmatrix} -4 & -1 & 3 \\ 2\lambda+2 & \lambda & -2\lambda-1 \\ \lambda-1 & 0 & 1-\lambda \end{pmatrix}$

**26.6 c)** .....  $\lambda \neq 1$

**26.6 d)** .....  $\frac{1}{1-\lambda} \begin{pmatrix} -1-\lambda+\lambda^2 & 1-\lambda & 2-\lambda \\ 1 & 0 & -1 \\ 1-\lambda^2 & \lambda-1 & \lambda-1 \end{pmatrix}$

## Fiche n° 27. Algèbre linéaire

### Réponses

27.1 a).....  $(3, -1)$

27.1 b).....  $(-1, 3)$

27.1 c).....  $(9/11, 2/11)$

27.1 d).....  $(-2, 4/5, 11/5)$

27.1 e).....  $(-1, 1/2, 1/2)$

27.1 f).....  $(0, 2, 4, 1)$

27.1 g).....  $(1/2, -\sqrt{3}/2)$

27.2 a).....  $2$

27.2 b).....  $1$

27.2 c).....  $1$

27.2 d).....  $2$

27.2 e).....  $2$

27.2 f).....  $1$

27.3 a).....  $2$

27.3 b).....  $2$

27.3 c).....  $3$

27.3 d).....  $4$

27.4 a).....  $\begin{pmatrix} 1 & 1 \\ 3 & -5 \end{pmatrix}$

27.4 b).....  $\begin{pmatrix} -5 & 3 \\ 1 & 1 \end{pmatrix}$

27.4 c).....  $\frac{1}{2} \begin{pmatrix} -19 & -43 \\ 9 & 21 \end{pmatrix}$

27.4 d).....  $\begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

27.4 e).....  $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

27.5 a).....  $\begin{pmatrix} -1 & -1 & 1 \\ 4 & 15 & 0 \end{pmatrix}$

27.5 b).....  $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

## Fiche n° 28. Équations différentielles

### Réponses

28.1 a) .....  $x \mapsto 56e^{12x}$

28.1 b) .....  $x \mapsto 6e^x - 1$

28.1 c) .....  $x \mapsto \frac{8e^{3x} - 5}{3}$

28.1 d) .....  $x \mapsto 9e^{2x} - 6$

28.2 a) .....  $x \mapsto e^{(6-x)/5}$

28.2 b) .....  $x \mapsto 1 - 2e^{-2x/7+2}$

28.2 c) .....  $x \mapsto \left( \frac{6}{\sqrt{5}} + \pi \right) e^{\sqrt{5}x} - \frac{6}{\sqrt{5}}$

28.2 d) .....  $x \mapsto \left( 12 + \frac{2e}{\pi} \right) e^{\pi x - \pi^2} - \frac{2e}{\pi}$

28.3 a) .....  $x \mapsto e^{2x}$

28.3 b) .....  $x \mapsto e^x$

28.3 c) .....  $x \mapsto 2e^{2x} - e^x$

28.3 d) .....  $x \mapsto (2 - 3i)e^x + (3i - 1)e^{2x}$

28.4 a) .....  $x \mapsto e^x$

28.4 b) .....  $x \mapsto 7e^{-x} - 5e^{-2x}$

28.4 c) .....  $x \mapsto \frac{4}{3}e^x - \frac{1}{3}e^{-2x}$

28.4 d) .....  $x \mapsto (2 - x)e^x$

28.4 e) .....  $x \mapsto (2 - x)e^{2-2x}$

28.5 a) .....  $x \mapsto \cos x + 2 \sin x$

28.5 b) .....  $x \mapsto e^{-x/2} \left( \cos \frac{\sqrt{3}x}{2} - \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}x}{2} \right)$

28.5 c) .....  $x \mapsto e^{-x} \sin(x)$

28.5 d) .....  $x \mapsto e^x \left( \frac{-1 + i}{2} e^{2ix} + \frac{1 + i}{2} e^{-2ix} \right)$

## Fiche n° 29. Séries numériques

### Réponses

29.1 a)....  $\boxed{\text{divergente}}$

29.1 b).....  $\boxed{2}$

29.1 c).....  $\boxed{2 + \sqrt{2}}$

29.1 d).....  $\boxed{\frac{1}{2 \times 3^9}}$

29.2 a).....  $\boxed{e}$

29.2 b).....  $\boxed{e^2 - 3}$

29.2 c).....  $\boxed{e^{\frac{1}{2}}}$

29.3 a).....  $\boxed{\frac{\pi^2}{6}}$

29.3 b)....  $\boxed{\text{divergente}}$

29.3 c)....  $\boxed{\text{divergente}}$

29.4 a).....  $\boxed{\frac{1}{12}}$

29.4 b).....  $\boxed{\frac{e}{e-1}}$

29.4 c).....  $\boxed{\frac{1-7i}{350}}$

29.4 d)...  $\boxed{\frac{-2-5\sqrt{2}i}{54}}$

29.5 a).....  $\boxed{1}$

29.5 b).....  $\boxed{\frac{1}{4}}$

29.5 c).....  $\boxed{\ln(2)}$

29.5 d).....  $\boxed{\frac{\pi}{4}}$

29.6 a)....  $\boxed{\text{divergente}}$

29.6 b).....  $\boxed{4}$

29.7 a).....  $\boxed{2}$

29.7 b).....  $\boxed{\frac{11}{4}}$

29.7 c).....  $\boxed{16}$

29.7 d).....  $\boxed{\frac{2e^3}{(e-1)^3}}$

## Fiche n° 30. Structures euclidiennes

### Réponses

30.1 a).....  $4 \ln 2 - 2$

30.1 b).....  $\frac{7}{12}$

30.1 c).....  $2 \sin(1) + \cos(1) - 1$

30.1 d).....  $\frac{1}{2}(e^2 - 1)$

30.2 a).....  $11$

30.2 b).....  $10$

30.2 c).....  $0$

30.3 a).....  $\frac{1}{6\sqrt{5}}$

30.3 b).....  $\frac{1}{5\sqrt{3}}$

30.3 c).....  $\frac{1}{3}$

30.4 a).....  $(1, 2\sqrt{3}(X - \frac{1}{2}))$

30.4 b).....  $(\sqrt{3}X, \sqrt{\frac{15}{43}}(4X^2 - 9X + 4))$

30.5 a).....  $\frac{1}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

30.5 b).....  $\frac{1}{5} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{pmatrix}$

30.5 c).....  $\frac{1}{11} \begin{pmatrix} 9 & -6 & 2 \\ -6 & -7 & 6 \\ 2 & 6 & 9 \end{pmatrix}$

# Fiche n° 31. Groupes symétriques

## Réponses

- 31.1 a)** ..  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 6 & 5 \end{pmatrix}$
- 31.1 b)** ..  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 5 & 1 & 3 & 4 \end{pmatrix}$
- 31.1 c)** ..  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 2 & 5 & 1 \end{pmatrix}$
- 31.1 d)** ..  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 6 & 5 & 3 & 4 \end{pmatrix}$
- 31.1 e)** ..  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 2 & 3 \end{pmatrix}$
- 31.1 f)** ..  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 1 & 5 & 4 \end{pmatrix}$
- 31.2 a)** .....  $(a b)$
- 31.2 b)** .....  $(c b a)$
- 31.2 c)** .....  $(7 2 5 3 1)$
- 31.2 d)** .....  $(a c b)$
- 31.2 e)** .....  $(2 1 5 4)$
- 31.2 f)** .....  $(1 2 7 5 3)$
- 31.3 a)**  $(1 7 4)(2 6 8 10)(3 9 5)$
- 31.3 b)** ..  $(1 3 10 6 4)(5 7)(8 9)$
- 31.3 c)** .....  $(1 7)(2 4 3 5 8)$
- 31.3 d)** .....  $(1 2)(3 4)$
- 31.3 e)** .....  $(1 4 6 2 3 5)$
- 31.4 a)** .....  $(1 4 2)(5 6)$
- 31.4 b)** .....  $\text{id}$
- 31.4 c)** .....  $(1 2 6 5 3)$
- 31.4 d)** .....  $(1 6 7 4)(2 5 3)$
- 31.5 a)** .....  $-1$
- 31.5 b)** .....  $1$
- 31.5 c)** .....  $1$
- 31.5 d)** .....  $-1$
- 31.5 e)** .....  $-1$
- 31.5 f)** .....  $1$
- 31.6 a)** .....  $-1$
- 31.6 b)** .....  $1$
- 31.6 c)** .....  $1$
- 31.6 d)** .....  $1$

## Fiche n° 32. Déterminants

### Réponses

- |               |            |              |                  |               |                          |
|---------------|------------|--------------|------------------|---------------|--------------------------|
| 32.1 a) ..... | $-2a^2$    | 32.2 c)..... | $227/336$        | 32.4 b) ..... | $6i - 12$                |
| 32.1 b) ..... | $6$        | 32.2 d)..... | $3\,919$         | 32.4 c).....  | $4/375$                  |
| 32.1 c).....  | $-5 + 6i$  | 32.2 e)..... | $7\sqrt{2} + 13$ | 32.5 a)....   | $x^3 + y^3 + z^3 - 3xyz$ |
| 32.1 d).....  | $20$       | 32.3 a)..... | $0$              | 32.5 b).....  | $-6 \ln^3(a)$            |
| 32.2 a).....  | $-2$       | 32.3 b)..... | $-40$            | 32.5 c)...    | $(y - x)(z - y)(z - x)$  |
| 32.2 b).....  | $9 \ln(2)$ | 32.3 c)..... | $0$              | 32.5 d).....  | $0$                      |
|               |            | 32.4 a)..... | $-4$             |               |                          |

## Fiche n° 33. Fonctions de deux variables

### Réponses

- 33.1 a) .....  $\{(x, y) \in \mathbb{R}^2, x - 1 \leq y \leq x + 1\}$
- 33.1 b) .....  $]0, +\infty[ \times ]0, +\infty[$
- 33.1 c) .....  $\{(x, y) \in \mathbb{R}^2, y \geq 0\} \setminus \{(0, 0)\}$
- 33.1 d) .....  $\emptyset$
- 33.2 a) .....  $\frac{\partial f}{\partial x}(x, y) = 2x + y$  et  $\frac{\partial f}{\partial y}(x, y) = 5y^4 + x$
- 33.2 b) .....  $\frac{\partial f}{\partial x}(x, y) = 2y \cos(2xy - y)$  et  $\frac{\partial f}{\partial y}(x, y) = (2x - 1) \cos(2xy - y)$
- 33.2 c) .....  $\frac{\partial f}{\partial x}(x, y) = (2xy, 2x)$  et  $\frac{\partial f}{\partial y}(x, y) = (x^2, -2y)$
- 33.2 d) .....  $\frac{\partial f}{\partial x}(x, y) = \frac{2}{1 + (2x + y)^2}$  et  $\frac{\partial f}{\partial y}(x, y) = \frac{1}{1 + (2x + y)^2}$
- 33.3 a) .....  $\frac{\partial f}{\partial x}(x, y) = -\sin(x - y)$  et  $\frac{\partial f}{\partial y}(x, y) = \sin(x - y)$
- 33.3 b) .....  $\frac{\partial f}{\partial x}(x, y) = \cos(e^{xy}) - xy \sin(e^{xy}) e^{xy}$  et  $\frac{\partial f}{\partial y}(x, y) = -x^2 \sin(e^{xy}) e^{xy}$
- 33.3 c) .....  $\frac{\partial f}{\partial x}(x, y) = y x^{y-1}$  et  $\frac{\partial f}{\partial y}(x, y) = x^y \ln x$
- 33.3 d) .....  $\frac{\partial f}{\partial x}(x, y) = \begin{cases} \frac{y^2(y^2 - x^2)}{(x^2 + y^2)^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{sinon} \end{cases}$  et  $\frac{\partial f}{\partial y}(x, y) = \begin{cases} \frac{2x^3 y}{(x^2 + y^2)^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{sinon} \end{cases}$
- 33.4 a) .....  $\sin(2t)$
- 33.4 b) .....  $\frac{2e^{4t} + e^{-2t}}{\sqrt{e^{4t} - e^{-2t}}}$
- 33.4 c) .....  $-72 \cos(4t) - 46 \sin(4t)$
- 33.5 a) .....  $\frac{\partial(f \circ \varphi)}{\partial u}(u, v) = \frac{1}{2} \frac{\partial f}{\partial x} \left( \frac{u+v}{2}, \frac{v-u}{2c} \right) - \frac{1}{2c} \frac{\partial f}{\partial y} \left( \frac{u+v}{2}, \frac{v-u}{2c} \right)$
- 33.5 a) .....  $\frac{\partial(f \circ \varphi)}{\partial v}(u, v) = \frac{1}{2} \frac{\partial f}{\partial x} \left( \frac{u+v}{2}, \frac{v-u}{2c} \right) + \frac{1}{2c} \frac{\partial f}{\partial y} \left( \frac{u+v}{2}, \frac{v-u}{2c} \right)$
- 33.5 b) .....  $\frac{\partial(f \circ \varphi)}{\partial r}(r, \theta) = \cos \theta \frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) + \sin \theta \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta)$
- 33.5 b) .....  $\frac{\partial(f \circ \varphi)}{\partial \theta}(r, \theta) = -r \sin \theta \frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) + r \cos \theta \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta)$